Elementary theory of the infinity-category of infinity-groupoids

Thesis director:

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Research project:

Homotopy theory is the branch of mathematics concerned with
space in its most abstract form, classically modelled by
topological spaces up to homotopy. Over the past 50 years, the
more abstract combinatorial notion of Kan complex became a
preferred model for homotopy theory, and they are now
interpreted as infinity-groupoids, and form together an
infinity-category. Higher category theory has thus become a
powerful language for homotopy theory, but also highly
technical. Currently, setting up the theory requires the
machinery of Quillen model categories. On the other hand, in
connection with Voevodsky's univalent foundations programme, it
was discovered recently that Constructive Type Theory, a fully
self-contained logical framework, is also a model for
infinity-groupoids, via an interpretation in Quillen model
categories. Type Theory and Quillen model categories share
certain strictnesses that do not have an intrinsic homotopical
interpretation. Taking the clue from Constructive Type Theory,
the present project aims at developing a purely synthetic
language for infinity-groupoids, independent of model
categories. It should implement more closely the
model-independent way homotopy theorists think about
infinity-groupoids, in strict analogy with Lawvere's elementary
theory of the category of sets (the categorical account of set
theory, which is much closer to mathematical practice and
intuition than Zermelo-Frenkel set theory), assuming classical
first-order logic.